In this section, we want to investigate the physical controls of water balance at the catchment scale. Initially we treat the catchment as a lumped system.

The catchment receives rainfall at the rate $p(t)$, loses water through evaporation, at the rate $e(t)$, and through runoff at the rate $q(t)$. The catchment water balance equation can then be written down (at any time scale) as:

$$\frac{dS}{dt} = p(t) - q(t) - e(t)$$

where $S$ is the storage of water in the catchment at any time $t$.

For simplicity, we start with annual water balance, ie., water balance over one whole year. We make the reasonable assumption that there is no carry over of water from one year to the next, thus at the annual time scale $dS/dt=0$, and we can write:

$$P = Q + E$$

where $P$, $Q$ and $E$ are the annual totals of rainfall, runoff and evaporation, respectively. In later sections, we will investigate how this partitioning will change as we go down to monthly, daily and hourly time scales.
2.1 Simple Overflow Bucket Model

We want to investigate the physical controls on annual water balance in a systematic manner – we start with simple models and gradually increase complexity to capture the basic features of the catchment’s response. We start with the simplest possible conceptualisation of a catchment – as a lumped bucket or store.

\[ p(t) \downarrow \quad e(t) \leq e_p(t) \]

Figure 1: Simple Overflow Bucket Model

Since we are interested in annual water balance, we assume rainfall and the radiation available for evaporation (converted to depth of water through the latent heat of evaporation, \(2.49 \times 10^9 \text{ J/m}^3\)) are uniform through the year at rates \(p(t) = p\) (eg, in mm/day) and \(e_p(t) = e_p\) (also in mm/day), respectively. We can also define the corresponding annual totals \(P = p \tau\) (in mm) and \(E_p = e_p \tau\) (also in mm) where \(\tau\) is the time period of 1 year (365 days).

Let \(S_b\) be the bucket capacity, thus storage in the bucket, \(S\), cannot exceed \(S_b\) at any time.

With respect to water balance equation, we assume that so long as water is available in the bucket it will evaporate at the potential rate, \(e_p(t)\), ie.,

\[ e(t) = e_p(t), \text{ for } S > 0 \]

\[ = \quad \text{ for } S = 0 \]

Similarly, we assume that there is no runoff so long as the bucket remains less than full. When the bucket is full and rainfall rate exceeds the potential evaporation rate, the bucket overflows and the runoff rate is given by \(q(t) = p(t) - e_p(t)\). This can be expressed as:

\[ q(t) = 0 \quad \text{for } S < S_b \]

\[ = p(t) - e_p(t) \quad \text{for } S = S_b \text{ and } p(t) > e_p(t) \]
This is why it is called the overflow bucket, and the runoff generated is called surface runoff.

Initially, we will assume that \( p(t) \) and \( e_p(t) \) are temporally and spatially uniform. We will now consider two cases: Case 1: \( E_p > P \) (dry or arid climate), and Case 2: \( P > E_p \) (wet or humid climate).

**Case 1: \( E_p > P \); also \( e_p(t) > p(t) \), for all \( t < \tau \) (Dry or Arid Climate)**

In this case the rate of input (rainfall) is smaller than the potential rate of removal – hence there cannot be any accumulation of water in the bucket, and under equilibrium or steady state conditions, \( S(t)=0 \), and \( q(t) =0 \) for all \( t < \tau \). Thus we have, on the annual time scale, all incoming rainfall being evaporated back, with no prospect of bucket overflow or runoff. Thus,

\[
E = P \quad \text{and} \quad Q = 0
\]

and

\[
E/P = 1 \quad \text{and} \quad Q/P = 0
\]

We see here that the actual evaporation rate is controlled by the amount of rainfall, regardless of how high \( E_p \) is. Thus this is a “water limited” system.

Another feature of this case is that \( S/S_b = 0 \) for all \( t < \tau \) there is no water storage, which means there is no prospect of vegetation growth, without vegetation the environment is adverse to other living things including humans.

**Case 2: \( P > E_p \); also \( e_p(t) < p(t) \), for all \( t < \tau \) (wet or humid climate)**

In this case, since we assume both rainfall and potential evaporation are uniform throughout the year, \( e_p(t) > p(t) \) for all \( t < \tau \). This means input rate is larger than removal rate and hence, in the long time (equilibrium), accumulation leads to bucket becoming full and remaining so for all time. Thus,

\[
q(t) = p(t) - e_p(t) \quad \text{for all} \quad t < \tau
\]

Thus,

\[
Q = P - E_p
\]

And

\[
E = E_p
\]
Here the upper limit to evaporation is the evaporation demand (or energy available for evaporation), and so clearly this is an “energy limited” system.

As stated above, in this case the bucket is full, i.e., \( S/S_b = 1 \) at all times and at all places – while water is full, energy is limited. If energy is too limited, even with plenty of water, no vegetation can survive, hence no animals and no humans can survive in this harsh environment.

2.2 Budyko Diagram/Curve

The character of annual water balance can be represented on the so-called Budyko diagram which presents the ratio \( E/P \) as a function of \( E_p/P \). \( E/P \) is a measure of annual water balance – it measures the way rainfall is partitioned into evaporation and runoff. On the other hand, the ratio \( E_p/P \) is a measure of the climate, and is called the dryness index (or index of dryness). Large \( E_p/P (>1) \) represents dry or arid climate, while small \( E_p/P (<1) \) represents a wet or humid climate. Thus the Budyko diagram encapsulates a major climatic control on annual water balance.

The results of the above simple model can now be presented on the Budyko diagram – this is presented below in Figure 1. Budyko carried out empirical analysis of the climate and water balance of a large number of catchments around the world, and showed that they all fitted a unique curve on the \( E/P \) vs \( E_p/P \) (Budyko diagram). The results of the simple model essentially produces two straight lines on the diagram:

\[
E/P = E_p/P \quad \text{for } E_p/P < 1 \\
E/P = 1 \quad \text{for } E_p/P > 1
\]

This can also be written as

\[
E/P = \min \{ 1, E_p/P \}
\]

These two straight lines happen to be upper envelopes for the empirically obtained relationship obtained by Budyko for natural catchments. Note that in our derivations of this relationship, no property of the landscape needed to be used – the results would have been the same regardless of the bucket capacity \( S_b \), for example; so far climate is the sole determinant of water balance.
According to our simple model, the whole world can be divided into “energy limited or wet or humid” areas and “water limited or dry or arid” areas. Humid areas are essentially swamps or cold and icy, and arid areas are just as desolate except they will be dry. Because of the simplicity of our model, these divisions are rather rigid, and there are no “grey” areas in between.

The above can be seen in the empirical Budyko curve – which departs somewhat from the envelope curves, and show that in all areas there is more runoff and less evaporation than are produced by our simple model. How can we explain these differences? These explanations are crucial because it is they that make the earth a more livable place for living things – plants, animals, humans.

The explanation has to be those factors that we have left out in our simple model – soils, vegetation, topography, and perhaps other aspects of climate that we have not included in our model so far. We pick up these additional controls in the forthcoming sections.